Spacetime and general relativity.

23.1 Four dimensional spacetime.

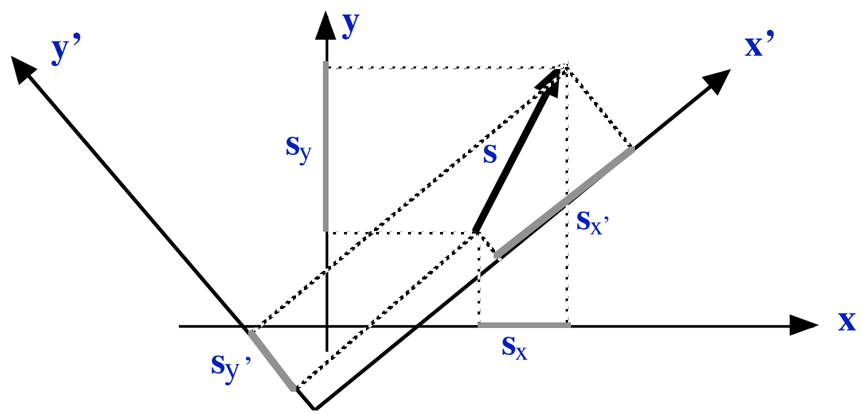
In fig. 21[xix] and 21[xx] we have seen that what one observer **O'** sees as two events occurring at the same time, another observer **O** will see happening at different times. The second observer also sees a different spatial separation. The changes can be dramatic at very high speeds. The universe looks different for different observers. We all see the world from our own frame of reference. The shape of our world depends on our speed relative to the things in it. Space and time are relative and it seems interchangeable.

Vectors.

In our everyday **3**D Euclidian world of absolute space and evolving time there are several constants or invariants. The laws of physics, the mass of an object, the uniform passage of time are examples. In this space the shortest distance between two points is a straight line. The displacement vector of one object from another is say **s**, then :-

$$s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2$$ this is the same to all observers.

Fig.23[i] A 2D invariant vector.



Time has no effect, nor does any relative motion under Galilean Transformations.

O and O' see the same vector from different angles and distances.

The components of the displacement **s** are different for the two observers, but the value of the magnitude is the same. The vector is invariant. In the 4D Relativistic world of spacetime, where the above are not true, there are still some invariants.

23.2 The relativistic interval. I

If we consider the quantity called **I** defined as

$$I^2 = [c\Delta t]^2 - \Delta s^2 \qquad \text{where } \Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$

and [**c∆t**] is the difference between the events on the 'time' axis and has the dimensions of length; and **∆s** is the separation in space.

If we then calculate the values for **I** and **I'** in fig.12[i] and 12[ii] we find that they are equal. Thus relativistic interval is invariant, or the same to all observers. It is not the same as displacement, but it is in a way the equivalent of displacement in the 3D world. It has four components, and the relativistic interval [**I**] is the separation of the events in spacetime.

[i] For Observer O'.

We are only measuring displacement in one direction, therefore

$$x = 2d' ; \; y = 0 ; \; z = 0 ; \; \Delta t' = 0 :$$

$$I'^2 = 0 - [2d']^2 = -4d'^2$$

The events E_F and E_R are separated by space only. The value of I^2 is negative which means that **I** is imaginary. The spacetime interval can be real, imaginary or in most cases complex. The real part of **I** is the time-like component and the imaginary part is the space-like component.

[ii] For Observer O.

$$\Delta t = {}^{2vd}/[c^2 - v^2] \qquad \text{and} \qquad \Delta s = 2dc^2/[c^2 - v^2]$$

$$I^2 = c^2\Delta t^2 - [2d]^2 = c^2({}^{2vd}/[c^2 - v^2])^2 - [\Delta S]^2$$

$$I^2 = -4c^2d^2/[c^2 - v^2]$$

The moving observer **O** will see a length contraction of the ship.

Thus $$d = d' \, [1 - v^2/c^2]^{1/2} \qquad \text{equation [3]}$$

Using equation [3] we get

$$I^2 = -4d'^2 = I'^2$$

Thus **I** is invariant under any Lorentz transformations of reference frames. This interval is similar to displacement in the 3D world, where the distance between two points is independent of time and the speed of the observer. That is, displacement, velocity, momentum or any vector is invariant under a Galilean transformation. What this means about **I** is; that the time and space between two events can be various amounts depending on the observer but **I** is always the same. It is possible for **Δs** and/or **Δt** to be zero. It is because it is invariant that a spacetime interval is somehow **more real** than the separate space and time components seen by any observer.

23.3 Four vectors.

Is the spacetime interval **I** a vector ? Yes, it is a vector with four components in the 4D spacetime world It has all the right properties. Relativistic Interval **I** is a four vector and has three spatial components and also a scalar time like component and it is invariant under Lorentz transformations. It is written **I** [x, y, z, ct]

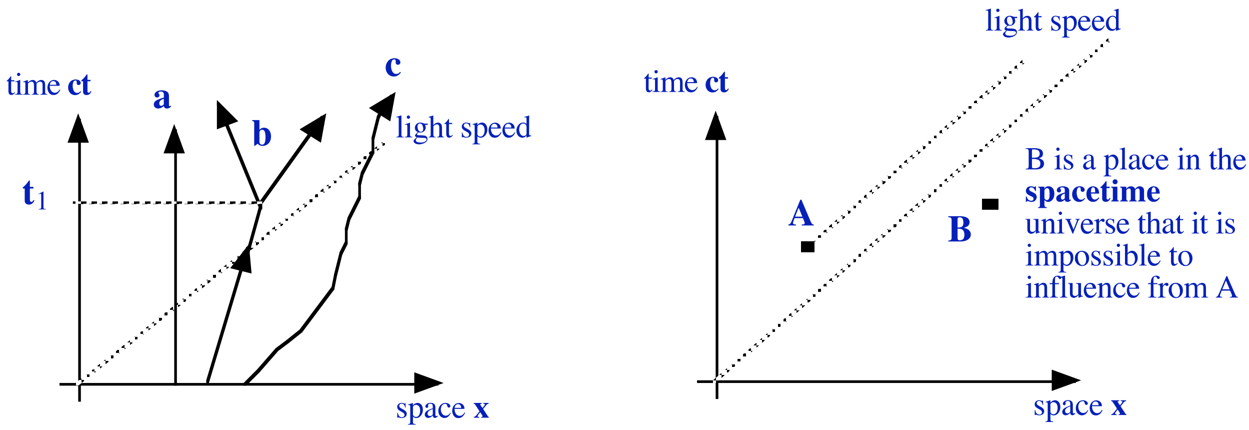
The magnitude of the interval is $$[c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2]^{1/2}$$

Is time the fourth dimension ? It must be considered as such since what we have now shown that what one observer sees as two events separated in time another may see as simultaneous and separated by distance.

23.4 World lines.

If we ignore the **y** and **z** directions and plot the position **x** against time **t** for an object, it is as shown below. The graph shows the 'history' of the particle. However, the time and space values are specific to a particular observer. The world line is different for other observers in different reference frames. The graph can be made simpler by multiplying the **t** - axis by **c**. The huge value for the speed of light would mean that the line showing the path of a light beam would be too close to the x - axis. This is really just the same as defining our units of time [or length] such that **c = 1**. No object can travel faster than the speed of light.

Fig.23[ii] World lines in spacetime.



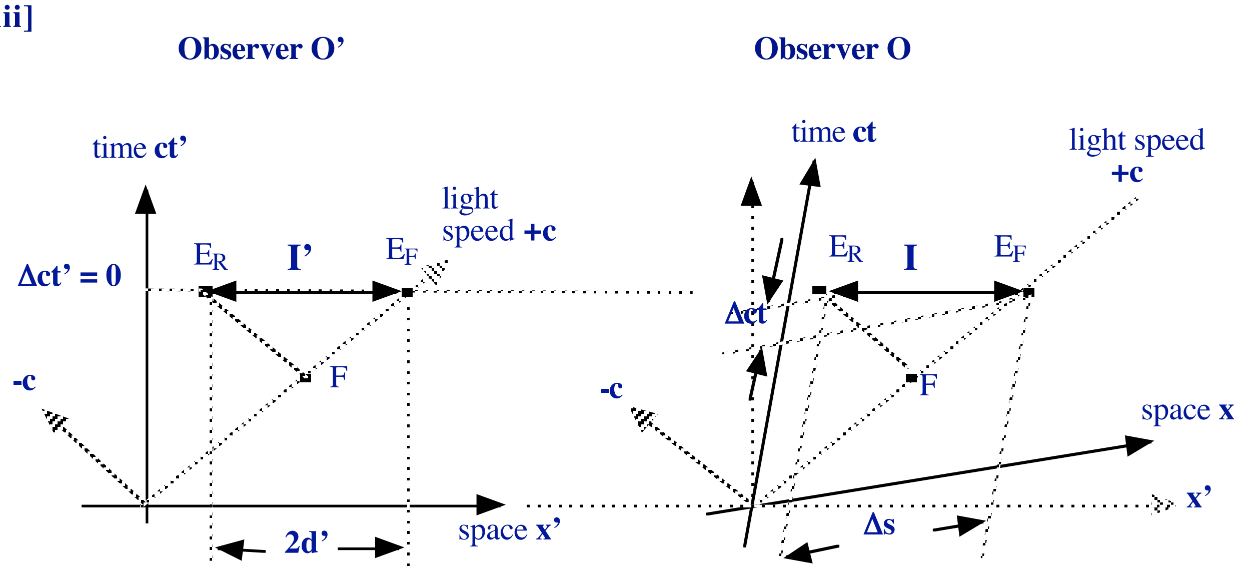
[a] A stationary particle
[b] A particle with constant speed, which disintegrates at time t_1
[c] A particle with a variable speed.

Anything leaving A cannot travel through spacetime faster than **c**. This means that the world line of an object is restricted to the cone formed by light travelling in opposite directions away from it. Of course, when we include time as a dimension, the real world now has **four dimensions** and it is not possible to draw them all. It is even impossible to truly visualise the situation and we have to rely on the mathematics.

Rather than draw different world lines on the same axes, the mathematics allows us to represent the effect of changing reference frame by changing the axes. This is the same as for conventional vectors as shown in fig. [12] where the one vector **s** is seen by two observers **O** and **O'**.

The mathematics shows that for a different observer moving at speed **v** relative to **O'** the effect of relative motion is to tilt the axes as shown below. The world line for light is the same on all axes and is along the centre line using **ct** for time units. The case of the flash of light in the previous section on simultaneity and spacetime intervals is used as an example in fig.20[xviii]. See if you can follow the diagram and relate it to the events in figs. 21[iii] and 21[iv]

Fig.23[iii]

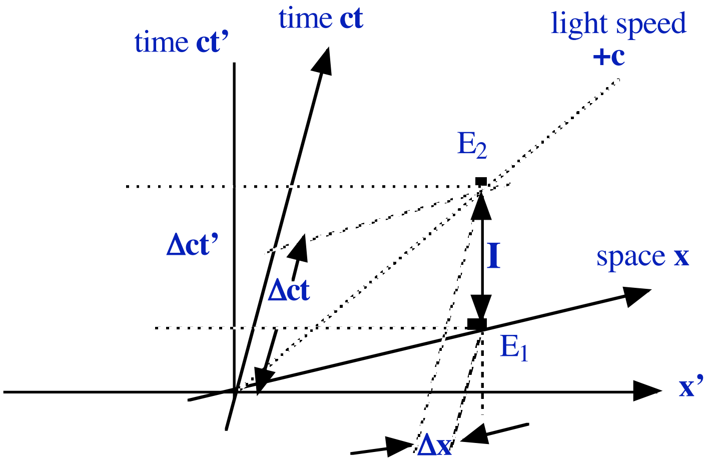


I = I' [measured on the respective axes]

2d' > 2d

The left hand diagram shows the light flash being emitted in opposite directions and reaching the ends of the spaceship simultaneously, as seen by **O'**. The right hand diagram shows the events F, E_R and E_F for the observer **O** outside the ship and for whom the ship is moving at speed **v**, in the x direction. Two events occurring at the same place, but separated in time, for **O'** can appear separated in space and time to another observer **O**.

Fig.23[iv] Time becomes space.



A pure time interval gains a spatial component.

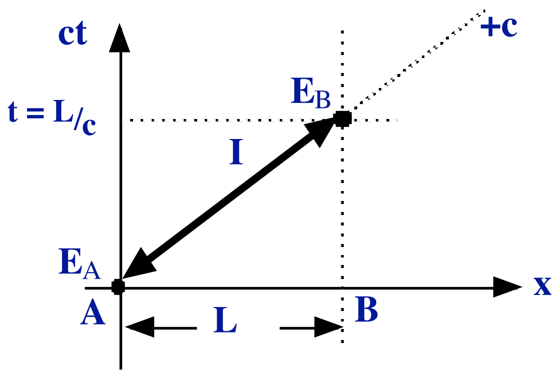
$\mathbf{I = \Delta ct'^2}$

$\mathbf{\Delta x' = 0}$

n.b. The scales are not the same. The axes are not really tilted when 'seen' by the observers, this is only a graphical representation of the effect of relative motion; a mathematical device. For an observer moving at near light speed the axes are almost together and the simultaneous events E_R and E_F for **O'** become widely separated in time and space.

Do not fall into the trap of thinking of **I** as the line drawn in the diagrams, or even the length of the line. It is a spacetime interval, defined mathematically, and very difficult to actually visualise. Consider figure 21[v] below where events E_A and E_B are the departure of light from **A** and the arrival at **B**, a distance $\mathbf{L_x}$ apart. The value of **I** is zero.

Fig.23[v] The Zero Interval.



$\mathbf{I^2 = [ct]^2 - L^2}$

$\mathbf{I^2 = L^2 - L^2 = 0}$

Clearly the interval **I** must not be thought of as the line, since in this case **I** = 0

Intervals for light are always equal to zero.

23.4 Four momentum.

In spacetime the interval **I** is the same to all observers. It has the same value when 'observed' or calculated by different observers. This is very much like a normal 3D vectors such as displacement and velocity. The components will change when looked at from different angles, but the magnitude remains constant.

There are other 4D spacetime quantities that are **invariant** under Lorentz transformations and one such quantity is called **four momentum** $\mathbf{p_\mu}$. This defined as consisting of the three spatial components of the momentum **p** and a scalar energy component, **E.** The magnitude of four momentum is :-

$\mathbf{p_\mu^2 = [E/c]^2 - p^2}$ where **E** is the energy and **p** is the relativistic momentum **p = mv**

The value E/c is used on the energy axis to equalise the size of units, but **E** is still energy.

Now $\mathbf{E = mc^2 = m_0c^2[1 - v^2/c^2]^{-0.5}}$

and $\mathbf{p = mv = m_0 v\,[1 - v^2/c^2]^{-0.5}}$

and a little simple mathematics gives us :-

$\mathbf{p_\mu^2 = m_0^2 c^2}$ which is obviously the same in all frames of reference.

Four Momentum $\mathbf{p_\mu}$ is **invariant** just like a 3D vector.

Four momentum has three spatial components of momentum and a scalar energy component. These components transform in the same way as the four vector **I** which has components **[x, y, z, ct]**. Four momentum $\mathbf{p_\mu}$ has components $[\mathbf{p_x, p_y, p_z, {}^E/c}]$, the components units are the same and the transformation equations for these are :-

$\mathbf{p_y = p'_y}$ $\mathbf{p_z = p'_z}$ [remember transverse momentum is invariant]

$$[E'/c] = \frac{[E/c] - vp_x/c}{[1 - v^2/c^2]^{0.5}} \qquad p'_x = \frac{p_x - \frac{v\,[E/c]}{c}}{[1 - v^2/c^2]^{0.5}}$$

These are exactly the same **form** as the Lorentz transformation equations for displacement and time if we write those using the time axis units of **ct**.

$$[ct'] = \frac{[ct] - vx/c}{[1 - v^2/c^2]^{1/2}} \quad \text{and} \quad x' = \frac{x - v[ct]/c}{[1 - v^2/c^2]^{1/2}}$$

In spacetime four vectors are as useful as normal vectors are in the 3D Euclidian world, if not more so. A four vector is a very useful quantity to know about because it enables us to move from one frame to another easily.

"Space of itself, and time of itself will sink into mere shadows, and only a kind of union between them shall survive."

Minkowski

23.5 Relativity and electromagnetic waves.

In the same year that Einstein published his special theory of relativity he also published a paper on the photoelectric effect. In this paper he proposed that light, and all EM waves, consisted of particles called photons. These are particles in the sense that they can be counted arriving at a position and they occupy a position in space. Thus light is actually carried by photons, which are packets of energy and therefore must have mass and momentum. However, the rest mass of a photon is zero. This means that the **four momentum of a photon is also zero**.

$\mathbf{p_\mu^2 = [E/c]^2 - p^2 = m_0^2 c^2 = 0}$ therefore $\mathbf{E/c = p}$

It was also Einstein who showed, in his 1905 paper on the photoelectric effect, that the energy of a photon was given by **:-**

$\mathbf{E = h\,f}$ and using the wave equation $\mathbf{c = f\,\lambda}$

then $\mathbf{p = {}^h/\lambda}$

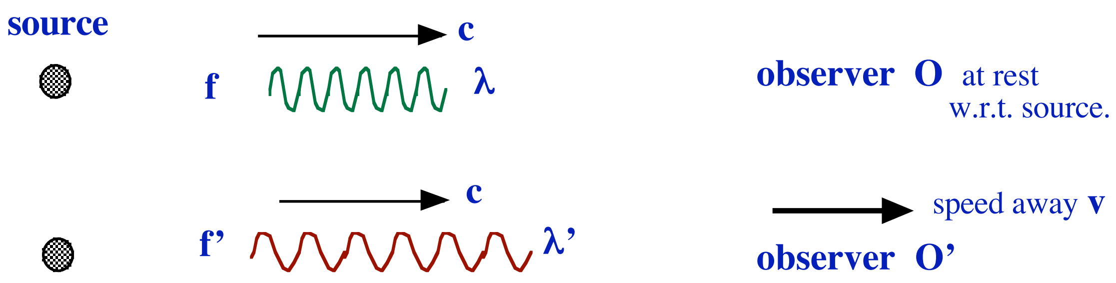
Compton showed that photons had momentum by colliding X-rays with atoms and measuring the energy of the scattered photons from their frequency and then showing that momentum was conserved. This is in fact true for particles as well, since all particles exhibit wave properties. The idea that photons can have wave and particle like properties led de Broglie to postulate that all matter exhibited wave properties. He assumed that the wavelength of a moving particle [with momentum] was

$$\lambda = {}^{h}/_{mv}$$

23.6 The Doppler effect and red shift.

For a particle with a non zero rest mass $\mathbf{m_o}$ the momentum is $\mathbf{p = mv}$ and when measured by observers in different frames it changes. When we transfer to a different frame the momentum of the photon **p** changes; therefore the energy **E**, the wavelength λ and frequency **f** change. This is true for the momentum of the photon as well. This is of course viewing the light while moving w.r.t. the source and is called the Doppler effect. The transformation equations derived above give the usual equation for the change in frequency as follows. Consider an observer **O** at rest w.r.t. a source of light with a frequency **f**. The photons have a momentum $\mathbf{p_x}$ and energy **E**. A second observer **O'** is moving at a speed **v** the same way as the light [in the positive x direction].

Fig.23[vi] Red shift.



$$c = f\lambda \qquad p_x = {}^{h}/_{\lambda} = {}^{hf}/_{c} \qquad p'_x = {}^{hf'}/_{c}$$

$$E = hf$$

The observer **O'** will see a longer wavelength, a reduced momentum $\mathbf{p'_x}$ and hence a lower frequency **f'**. Using the transformation equations for four momentum.

$$p'_x = \frac{p_x - \dfrac{v\,[E/c]}{c}}{[1 - v^2/c^2]^{0.5}} = hf'/c = \frac{hf/c - v\,[hf/c^2]}{[1 - v^2/c^2]^{1/2}}$$

$$f' = f\,\frac{[1 - v/c]}{[1 - v^2/c^2]^{1/2}} \qquad f' < f$$

In this case the observer **O'** is moving away from the source**.** The frequency observed is less than that seen by **O** and [he] and sees a **redder** light . Actually there is no way of telling whether he is moving away from the source or the source is moving away from him. Therefore the change in frequency [**red shift**] is the same for the case of a stationary observer and a moving source, and it is not necessary to derive the expression. It is this red shift in the light from the distant galaxies that tells us that they are receding and that the universe is expanding.

If [he] goes in the opposite direction, towards the waves, the speed **v** is negative and he will see a higher frequency [bluer] light.

n.b. These equations for EM waves differ from those for the doppler effect in sound waves due to the constancy of the speed of light to all observers. That is time passes slower for the moving observer.

Comparison of EM waves with Sound Waves.

In the case of an observer **O'** moving in the same direction as the sound waves at speed **v** the formula is as follows: The speed of sound in air is $\mathbf{v}_S$

Fig.23[vii] **Sound and EM waves.**

Motion	Sound Waves	EM Waves
Observer moving away from [-] or towards [+] the source	$f' = f\,[1 \mp {}^{v}/_{v_s}]$	$f' = f\,\dfrac{[1 \mp {}^{v}/_{c}]}{[1 - v^2/c^2]^{1/2}}$
Source moving towards or away from the observer.	$f' = \dfrac{f}{[1 \mp {}^{v}/_{v_s}]}$	No change to formula

You can see that the only difference is in the denominator. This factor appears for light due to the time dilation that **O'** experiences. He will experience less time and therefore see a higher frequency.

The big difference in the case of sound is that, when the source is moving, the expression is different to the case for a moving observer. This is because the speed of sound is not constant w.r.t. the observer, but constant w.r.t. the air. This is, of course, the way that light and the ether were thought of, until the result of the Michelson and Morley experiment showed that the speed or light was in fact constant w.r.t the observer not the ether.

We have seen the consequences of a simple experimental fact on the laws of physics. Namely the constancy of the speed of light. Another simple but crucial breakthrough by Einstein was to enable accelerating frames to be included. These are called '**non inertial frames**'. A **very brief** section on this follows as the mathematics is much more complicated.

23.7 The general theory of relativity.

Einstein's second great work on motion and gravity was published in 1915. This brought together non inertial frames and gravity in a theory that has to this day withstood the most searching examination by experiment. His great insight was to postulate the **Principle of Equivalence**. This states that :-

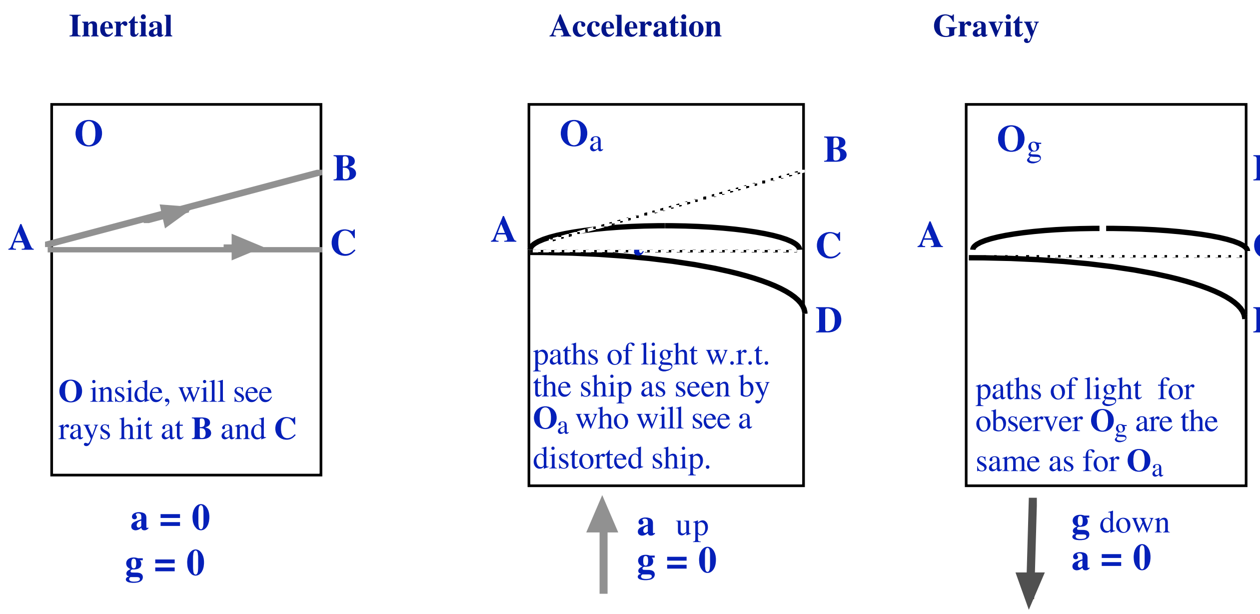
> ***" It is impossible to determine if one is in a state of accelerated motion or in a gravitational field."***

This has been discussed in chapters 12 and 13. Falling freely is indistinguishable from zero acceleration. Conversely, if an observer is stationary in a gravitational field, the force of gravity is indistinguishable from the force felt when accelerating. Thus, if the effects are completely indistinguishable, then light must behave the same way in both.

The curvature of spacetime by gravity.

This means that for an observer standing in a gravitational field the effect of gravity on light is indistinguishable from the effect of observing the light from an accelerating frame of reference. Look at the two rays in each situation below. The inertial observer watching would see the rays hit at **C** and **D**, instead of **B** and **C**. The observers $\mathbf{O_a}$ and $\mathbf{O_g}$ in the ship must also see them hit as shown.

Fig.23[viii] Paths of light in different situations.



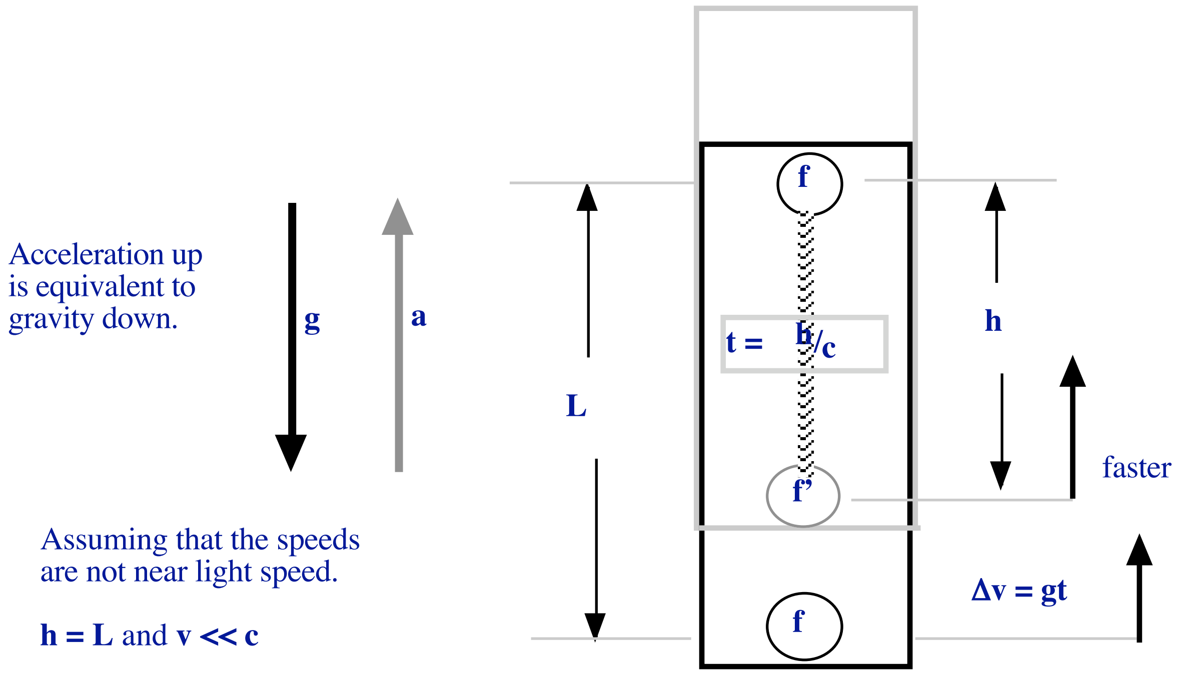
What is more interesting is to look at the rays reversed. Imagine light moving from **B, C** and **D** towards **A**. In fig.[i] **O** will see **B** and **C** where they are. In fig.[iii] $\mathbf{O_g}$ will see **D** as directly opposite **A** and **C** somewhere near **B**. The ship is distorted by gravity. If they can also measure the times taken, $\mathbf{O_a}$ and $\mathbf{O_g}$ will agree on the value and in fig.[i] the observer **O** will record a **greater** time interval than the other two. This is explained below.

23.8 The effect of gravity on time.

[i] In an accelerating frame of reference [or in a g field].

Imagine two Caesium atomic clocks at the front and back of a rocket ship, that is accelerating as shown. This is equivalent to being in a gravitational field acting downwards. The clocks have a frequency that is used to measure time and the rest frequency observed is **f**. Imagine that the front [top] clock emits a EM wave at this frequency towards the rear [bottom] clock.

Fig.23[ix] Gravity and time.



Using the above $v = \Delta v = -gt$ $\quad t = {}^{h}/_{c}$ $\quad v = {}^{-gh}/_{c}$

Substituting these values in the equation for the doppler effect, the frequency of the top clock, observed at the bottom of the ship is

$$f' = f \frac{[1 - {}^{v}/_{c}]}{[1 - v^2/c^2]}{}^{1/2} \qquad f' = f\ [1 + {}^{gh}/_{c^2}\ ]$$

The frequency is higher, which means that the top clock goes faster and we can

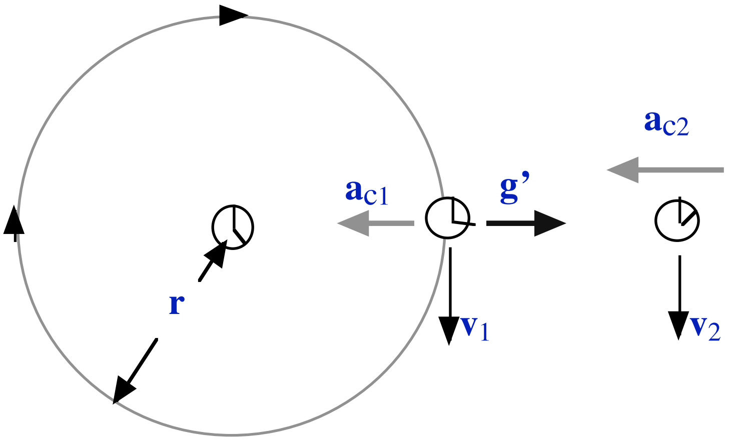
write $t' = t\ [1 + {}^{gh}/_{c^2}\ ]$

Time passes slower low down in a gravitational field.

[ii] Clocks in a rotating frame of reference.

These are also non inertial frames since there is a centripetal acceleration. Acceleration in rotational motion is also indistinguishable from gravity. The artificial gravity **g'** in a rotating space station is a good example of the application of the principle of equivalence. According to Special Relativity, a moving clock going round in a circle goes slower than a clock that is stationary.

Fig.23[x] Clocks in a Rotating Frame.



Time is slowed by the acceleration.

$a_{c2} > a_{c1}$

The artificial gravity is indistinguishable from the real thing.

If we consider the stationary clock to be at the centre of the circle and rotating with the moving clock; we can see that in the rotating frame there is now no relative motion. The outer clock must **still** go slower and it must be the acceleration that causes this, but accelerating frames are indistinguishable from gravity. **This means that gravity will alter time.** We can use the example of circular motion to investigate this effect.

The centripetal acceleration is $a_c = v^2/r = g'$ $v^2 = rg'$

Special Relativity gives us $t = \dfrac{t'}{\sqrt{1 - v^2/c^2}}$

Therefore if we put ourselves in the rotating frame the clocks must still differ by the same amount.

$$t = \frac{t'}{\sqrt{1 - rg'/c^2}} = t' \, [\, 1 + rg'/2c^2 \,]$$ ignoring further terms.

Therefore $t' < t$ and the clock in the centre goes faster. However, remember the artificial gravity **g'** acts **outwards** in the rotating frame. This means that as you enter the Earth's gravitational field your clocks will go slower. This means that the lower a clock is in a gravitational field the slower time passes. Actually the mean value for **g'** over the radius **r** is $g'/2$ and the approximate effect of a uniform

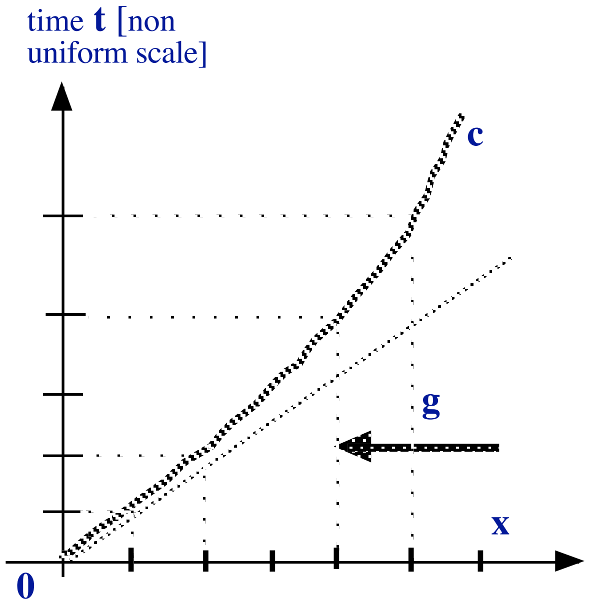
gravitational field **g** over a 'height' **h** is :-

$$t_h = t_o \, [\, 1 + hg/c^2 \,]$$ Which is in agreement with the first derivation.

23.9 Curved spacetime.

Since gravity bends light, this means that time and length will be altered by gravity. Spacetime is warped or bent and the spacetime interval **I** between two events is affected by gravity.

Fig.23[xi] The curvature of spacetime by gravity



The speed of light is constant.

There is a gravitational field **g** acting towards the origin.

Hence as a beam of light moves [climbs] in the **x** direction, time slows down and the path for light becomes a curve in spacetime.

This is what is meant by the curvature of spacetime.

For any particular observer space is warped by gravity and the shortest distance between two points is not a straight line, in the Euclidian sense of the word. The shortest distance is always the path that light takes, this is called a **geodesic**.

This must be true or it would be possible to travel from say **P** to **Q** by a shorter route at slightly below light speed and get to **Q** before the light. It would mean, in effect, that you could travel back in time and alter the past. This has to be considered impossible. In fact this must be impossible in all frames therefore observers **all** see the light paths as the shortest between the two respective points.

The bending of light by gravity was observed in 1919 and measured with reasonable accuracy. The amount of bending was found to be very close to the amount predicted by Einstein's theory. Atomic clocks have been flown around in circles and then compared to stationary Earth based clocks. The accelerating clocks were found to be behind the ground based clocks.

If the gravitational field is strong enough or high enough, then the slope of the light path becomes 'vertical' and the light can climb no higher in the gravitational field. This is the property of curved spacetime that leads to the prediction of **black holes**.

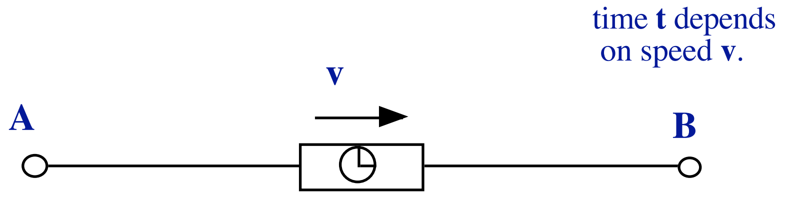
23.10 Motion in spacetime and 'proper time'.

Gravity and relative motion **both** affect clocks. Since various observers of any two events separated in space will record different time intervals, is there any time with particular significance? **Proper time** is that registered by a clock that is with the observer throughout any journey in spacetime. We will see that this time does give us useful information regarding motion in spacetime :-

[i] Uniform motion.

Consider a trip from **A** to **B** in a straight line and in the absence of gravity. If we stipulate a time for the journey as measured by a set of clocks that are stationary w.r.t the points A and B., which type of motion between **A** and **B** will give the **greatest** proper time ?

Fig.23[xii] Uniform motion



It must be a uniform speed, since if we go faster than this for a while our clock slows down. If we then slow down, to finish the trip at the correct time, our clock does not compensate enough, since the effect on time depends on the **square** of the speed. Therefore the motion that has the greatest proper time is **uniform motion.**

Thus the condition for maximum proper time corresponds to Newton's first law or the law of inertia. Although we do not seem to have any physical basis for such a relationship, it is a property of spacetime. One can think of this as uniform motion through time, in that the clock will not 'accelerate' in the sense of time passing faster or slower. Uniform motion is truly uniform in all four dimensions of spacetime.

[ii] Accelerated motion.

If one is accelerating 'through space', one also accelerates 'through time'. Of course the traveller would never feel or observe time passing faster or slower. It is the inertial observer that sees the changes in the voyager's clock. If the acceleration is positive the clock slows down and if it is negative the clock speeds up. Therefore the traveller is truly accelerating in four dimensions. We must remember that acceleration is indistinguishable from being in a gravitational field.

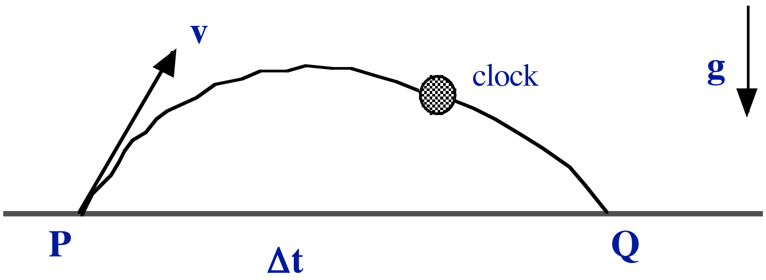
[iii] Motion in a gravitational field.

An astronaut in a ship that is stationary in a gravitational field at a position where there is an 'acceleration due to gravity', will find that time measured by [his] clock passes faster than for a clock outside the field, but at a constant rate. The effect of the field is to bend or warp spacetime and this means that the time dimension is bent as well as the space dimensions. If the ship moves through the field to a position where **g** is different the rate of passage of time will change or 'accelerate' while he is moving.

Now consider motion in a uniform gravitational field **g**. The simplest case would be a path straight up and down. If we stipulate the time that the object must take to get from A to B and back to A; what type of motion will give the greatest proper time for the trip ? The clock will go faster as it rises in the gravitational field and it will also go faster as it slows down. Therefore it is better to stay high as long as possible.
However, not too long or the high speed necessary to get back down in the stipulated time will slow the clock down by a very large factor. If we do the maths we find that the resulting motion is exactly that of a projectile moving freely under the action of gravity. The required initial upwards velocity and the resulting motion is exactly that calculated using Newton's laws of motion.

Lastly, imagine a projectile near the surface of the Earth carrying a small clock. If we stipulate the time for a trajectory from say **P** to **Q** and then consider how the speed and trajectory might vary and still take the same time for the trip, as measured by a stationary observer near to **P** and **Q**, an interesting fact emerges.

Fig.23[xiii] A projectile in the Earth's field.



Of course, if the rate of clocks was unchanged then there would be an infinite number of ways to get from **P** to **Q** in the set time. Even allowing for the slowing down of a clock with speed and the speeding up of the clock as it gets higher, there are still an infinite number of ways of completing the trip in the stipulated time, as measured by clocks stationary w.r.t. the ground. There is only one way of arriving at **Q** in the maximum **proper time**.

If the motion was uniform from **P** to **Q** horizontally one might expect a maximum proper time, but we can get a greater time by rising in the gravitational field. The clock goes faster at altitude. This means going faster initially since the distance travelled is now greater but the motion that gets the projectile to **Q** in the stipulated time [and with the maximum proper time] is exactly that given by Newton's laws. It is a parabola.

23.11 Einstein's laws of motion.

Therefore it seems that we can replace the notions of force and inertia with the idea that all motions in spacetime under the action of gravity are those with the greatest proper time. These are **Einstein's laws of motion.**

1. Motion in spacetime either in the absence of gravity and/or in free fall in a gravitational field is always along paths that have a **maximum proper time**.

2. Spacetime is curved by large masses. The **force** of gravity is replaced by the curvature of spacetime caused by large masses.

Other forces, such as the electromagnetic are not included in this description and must be added when they occur.

23.11 Conclusion

Today the story of spacetime, forces and inertia is still not complete. There is still argument about the contribution of space to inertia and also the interaction of the distant galaxies or the rest of the universe with objects. The space between the galaxies [or any two objects] is **not nothing.** It is space, and it was not always there. Space is growing; it can be stretched and bent by gravity, electromagnetic fields and nuclear fields. It is full of amazing high speed virtual particles that mediate or 'carry' the four fundamental force interactions. It is different to observers who have a relative velocity. The **vacuum** of space has properties and it is most certainly not nothing. Today, we believe that **nothing** was before the universe began.

"When one knows that the great void is full of ch'i one realises that there is no such thing as nothingness"

Chang Tsai

"the enormous energy content of the vacuum"

David Bohm

Curved space, time travel, black holes, seem like fiction, but they are not. Maybe warp drives will be invented one day. Maybe worm holes do exist and we will reach the far side of the universe in a small jump through a worm hole. I hope that you were able to follow most of the arguments and the mathematics, which can get laborious at times. It is important that you realise that quantities like force, energy, mass and others, even time, are **constructs** of the human mind and not things that exist in their own right. We as observers put our interpretation on the 'things' we see and the way we interpret our observations changes. The concepts of the Greeks were not wrong they were only limited in their extent and at the time were held to be as true as our beliefs are today.

There is probably only one reality but we, as observers with limited senses, cannot visualise it in its entirety. Different parts of it affect us in different ways. We **see** light, **feel** heat, **touch** matter, **perceive** distance, **experience** forces and these events occur **in** time. As Einstein shows us our perception of the passage of time and of space is very limited, it can be very different for different observers. In a very real sense time does not exist independent of the events that happen. We all personally experience time, but to the physicist we exist in a four dimensional spacetime, in which past, present and future are **just there**. Space is relative and does not exist independent of the objects in it or the observer. It is the events that define spacetime not visa versa and the observer 'sees' these from various 'angles'.

It is unfortunately true that today the physicist has to rely on mathematics to describe nature. However as a tool it is very reliable and great advances have been made. There is almost always a physical phenomenon or explanation for the relationships that the mathematics throws up. We will see in the chapter on quantum physics that the observer's role becomes even more important. The photon, and even things we consider as real, have to be described by wave functions that, in a sense, do not exist until the act of observation. Perhaps it is more correct to say that the wave function is not observable and collapses at the moment of observation, but that is another story.

Scientists are always trying to make the picture simpler and also more general. Newton's laws of motion and gravity are not wrong as Einstein's theories of relativity encompass them as a special cases and Aristotle's theories are a simplification of motion with a constant frictional resistance. The picture today is bigger and we are closer to the G.U.T. of everything. However, the answer is not **forty two**.

"There is such a thing as a passionate desire to understand, just as there is a passionate love for music. Without it, there would be no natural science, and no mathematics."

A. Einstein

"The world may be an amazing place, but warped spacetime is something else"

P.W.S. [1997]